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No. 1.

PRICE 4d.

A COURSE
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GEOMETRICAL DRAWING;
OR,
PRACTICAL GEOMETRY, PLANE AND SOLID:

INCLUDING THE
CONSTRUCTION AND USE OF SCALES,
ORTHOGRAPHIC, HORIZONTAL, AND ISOMETRIC PROJECTION, AND THE
THEORY OF SHADOWS.

DESIGNED FOR ARTISAN AND ENGINEERING STUDENTS,
AND FOR THE USE OF SCHOOLS.

BY C. SPRIGGS, MECHANICAL ENGINEER.

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P R E F A C E.

THIS publication is intended to supply students of science classes and others with a cheap yet comprehensive text-book on Engineering Drawing.

As Geometry is the basis of all drawing, this volume is devoted to that subject, and is divided into two branches or parts.

PART FIRST treats of Practical Geometry of the Plane, such as the division of lines; construction of right-line figures; tangency of circles and of circles and lines; construction of scales, plain and diagonal; of various curves; of figures equal to given areas, and similar to given figures; and, generally, the construction of such problems as are of utility to all who are engaged in the mechanical and decorative arts.

PART SECOND treats of Practical Solid and Descriptive Geometry, such as the projection of any point or line on two co-ordinate planes; the representation of simple solids, such as prisms, pyramids, cylinders, cones, and spheres, on these planes by means of their projections, or solids formed by their combinations; and all that relates to the representation of lines or planes in any position with respect to the horizontal and vertical planes of projection (as represented by the paper).

In a work of this kind great originality is not to be expected, so far as the geometrical figures themselves are concerned: the *instructions*, however, for working out the various problems are entirely re-written; and, with regard to practical utility, I have endeavoured to adapt the work to the comprehension of such as have not had the advantages of "mathematical instruction." For this purpose I have introduced some new features, which, from my experience as a teacher, I have thought advisable. They are the following:—

The application of the properties of "Plane Loci" in the construction of many of the Problems; thus showing their limits, and, at the same time, rendering them capable of being understood by the tyro in Theoretical Geometry.

The distribution of Definitions and Theorems over the whole work; thus impressing on the mind of the student the "principles" or "truths" on which the Problems are constructed.

An extensive selection of Miscellaneous Questions for practice, arranged in such a manner as to enforce the necessity of the student's constantly referring to what he has already learned.

With these new features it is hoped that the value of the work may be thereby enhanced, and that it may be the means of enabling those for whom it has been designed to acquire that knowledge, which they can apply in their daily occupation; with benefit to themselves and those with whom they may come into contact.

C. SPRIGGS, M.E.

MANCHESTER, September, 1870.

GEOMETRICAL DRAWING.

PART FIRST.

PRACTICAL PLANE GEOMETRY.

INTRODUCTION.

GEOMETRY is that branch of mathematical science which treats of magnitude in its threefold properties (length; length and breadth; length, breadth, and thickness), and of the relations which exist between them. By its aid the forms of objects, superficial and solid, can be represented, so as to convey the most correct ideas of their localities, bulk, and proportions. Geometry is divided into two branches, called Theoretical and Practical : the first comprises the *principles* of the science ; the second, the application of those *principles* to useful purposes of every-day life. Both the theory and practice of Geometry will mutually assist each other, yet they may be studied separately with great advantage. The present work has been designed chiefly to guide practical men, who have not much time to devote to the theory of the subject ; and to bring within their reach that which may be made available in the various departments of mechanical, architectural, and decorative arts.



FIG 1'

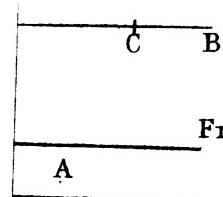


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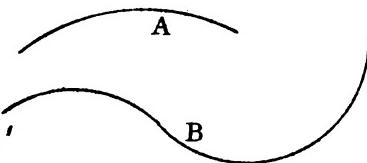


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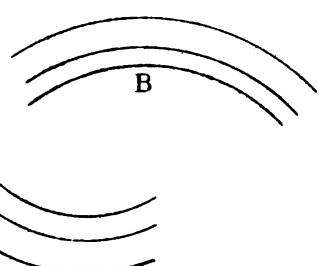
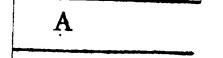


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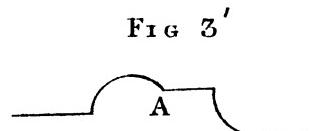


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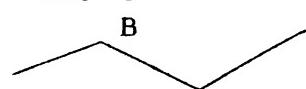


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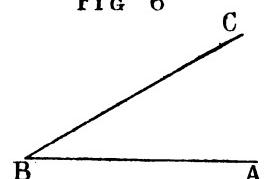


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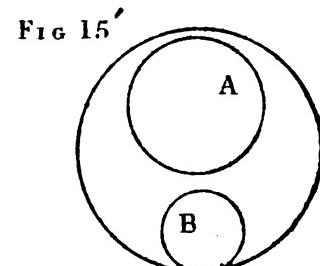
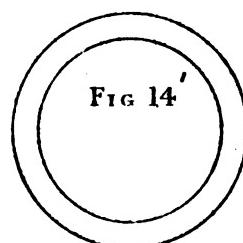
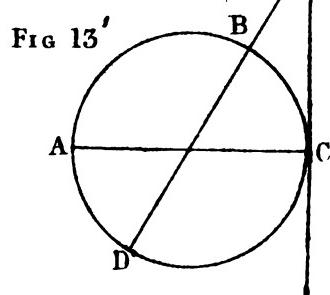
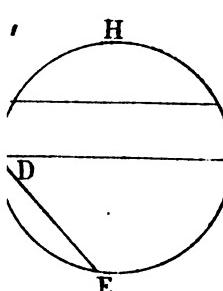
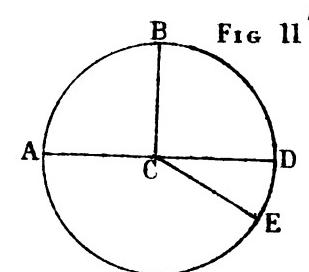
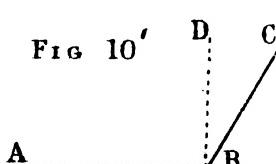
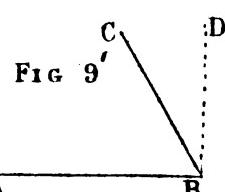
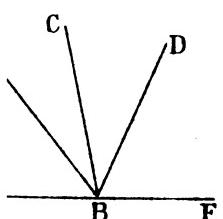
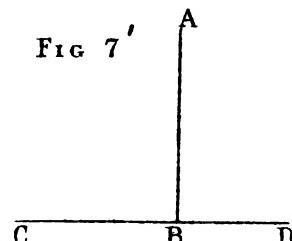


PLATE I.

DEFINITIONS AND EXPLANATION OF TERMS.

- 1 DEF.—A point is a term used to denote or express a position.
- 2 DEF.—A line is length without breadth.
Lines may be drawn in any direction, and are called *straight* or *right lines*, *curve* or *curved lines*, *mixed* or *compound lines*, *irregular* or *broken lines*, as they correspond to their definitions.
- 3 DEF.—A straight or right line is that which lies evenly between two points.
It is of course the shortest line that can be drawn between two points, as from **A** to **B** (Fig. 1'). Hence, the extremities of a line are points, and the intersection of lines are also points.
In speaking of points, we distinguish one from another by using letters of the alphabet to mark their position; and so also with regard to lines, to mark their position or extent, or both. A single letter will express or denote a point, but two are mostly used to express a straight line: thus, in Fig. 1', if we put **A** at one end of the straight line, and **B** at the other end, the points which are the extremities of it are simply called the "points" **A** and **B**. And the line is called the "line" **A B**.
A part of a line is called a segment of it: thus, in Fig. 1', **A C** is called a "segment" of **A B**, and **C B** is another "segment" of it.
NOTE.—When the word "line" is used by itself in the following pages, a *straight line* is to be understood.
- 4 DEF.—A curve or curved line is such that no part of it is straight between its extremities; and may be either regular or irregular: as **A** and **B** (Fig. 2').
- 5 DEF.—A mixed or compound line is composed of straight and curved lines: as **A** (Fig. 3').
- 6 DEF.—An irregular or broken line is composed of two or more straight lines: as **B** (Fig. 4').
- 7 DEF.—A superficies (or surface) is that which has length and breadth, but no thickness.
A good representation of a surface is that of a shadow; its length and breadth can be measured, but it has no depth or thickness.
- 8 DEF.—A plane surface is that in which any two points being taken, the straight line between them lies wholly in that surface.
A surface is plane when a straight edge is in contact with it at every point, in whatever position it is applied to it. Any other surface is called a *curved* surface.
- 9 DEF.—Parallel lines are such as lie in the same plane, and which, though produced ever so far both ways, would never meet.
Parallel lines are of course those which are everywhere equally distant from each other, and may be either straight or curved: as **A**, **B**, and **C** (Fig. 5').

PRACTICAL PLANE GEOMETRY.

- 10 DEF.—An angle is the inclination or opening of two lines which meet in a point : as Fig. 6'.
- 11 DEF.—The point where the two lines meet is called the angular point or vertex.
An angle is usually denoted or expressed by three letters, of which the letter at the angular point is always placed between the others : thus, we say the angle **A B C** (Fig. 6'), or **C B A**. When there can be no doubt as to what angle is intended to be expressed, it may be so expressed by a single letter, placed at its vertex.
- 12 DEF.—When a straight line **A B** (Fig. 7) meets another straight line **O D**, so as to make the adjacent angles **A B C**, **A B D** equal to each other, each of the angles is called a *right angle*; and the straight lines are said to be *perpendicular* to each other.
It therefore appears from the above definition, that if any number of lines meet a line in the same point, and on the same side of it, all the angles made by these lines are together equal to *two right angles*: thus, if the lines **A B**, **C D**, and **D B** (Fig. 8'), meet the line **E F** in the point **B**, the angles **A B E**, **C B A**, **D B C**, and **F B A** are together equal to *two right angles*.
- 13 DEF.—Every angle *less* than a right angle is called an *acute angle*; and every angle *greater* than a right angle is called an *obtuse angle*: as shown in Figs. 9' and 10'.
- 14 DEF.—A plane figure is a portion of a plane surface enclosed or contained by one or more lines or boundaries.
- 15 DEF.—A circle (see Fig. 11') is a plane figure bounded or contained by one line **A B E**, which is called the *circumference*; and is such that all lines drawn from a certain point (**O**) within it to the circumference are equal.
The point **O** from which the lines may be thus drawn is called the *centre*; and the distance from the centre to the circumference is called the *radius*: as **O A**, or **O B**, or **O E**.
- 16 DEF.—Every line which, like **A B** (Fig. 11'), passes through the centre, and is terminated both ways by the circumference of a circle, is called a *diameter*.
- 17 DEF.—Any portion of the circumference of a circle, such as **F H G** (Fig. 12'), is called an *arc*; and the line **F G** which joins its extremities is called a *chord*.
- 18 DEF.—Any portion of a circle cut off by a chord is called a *segment*: as the portion of the circle contained by the arc **F H G** (Fig. 12') and the chord **F G**. When the segment is equal to half the circle it is called a *semicircle*: as **A H B** or **A E B** (Fig. 12').
- 19 DEF.—Any portion of a circle contained by two radii and the intercepted part of the circumference is called a *sector*: as **A C E** or **D O E** (Fig. 11'). When the two radii are at right angles, the sector is called a *quadrant*: as **A B C** or **B C D** (Fig. 11').
- 20 DEF.—A *tangent* to a circle is a line which touches the circumference without cutting it: as **T O S** (Fig. 13). And the point where the line touches the circumference is called the *point of contact*.
- 21 DEF.—If a diameter or the chord of a circle be produced, the line thus produced is called a *secant*: as **T B D** (Fig. 13).
- 22 DEF.—*Concentric circles* are circles within circles, described from a common centre: as Fig. 14'.
- 23 DEF.—*Eccentric circles* are those which are not described from a common centre: as in Fig. 15'.

PRACTICAL PLANE GEOMETRY.

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24 DEF.—If a point vary its position according to some determinate law, it will trace a line which is said to be the *locus* of the point.

As an illustration of a locus, consider that of a point which is always at the same distance from a given point. It is evident that the line traced by the point will be the circumference of a circle (see Def. 15), whose radius is equal to the given distance.

25 A THEOREM is a truth which requires to be demonstrated by a process of reasoning.

26 A PROBLEM is something proposed to be done: as the construction of a figure, or the solution of a question.

27 AXIOMS are truths so evident as to require no demonstration. The Axioms are as follow:—1. Things which are equal to the same thing, or equal things, are equal to one another. 2. If equals be added to equals, the wholes are equal. 3. If equals be added to unequals, the sums or wholes will be unequal. 4. If equals be taken from equals, the remainders will be equal. 5. If equals be taken from unequals, the remainders will be unequal. 6. Only one straight line can be drawn from one point to another. 7. Two straight lines cannot be drawn through the same point parallel to another straight line without coinciding with each other. 10. All right angles are equal to one another. 11. Equal circles have equal radii.

28 A POSTULATE signifies something supposed or assumed to be practicable. As, for example, 1st, that a line may be drawn from any one point to any other point; 2ndly, that a line may be produced, that is, lengthened or continued at pleasure; 3rdly, that a circle may be described from any centre, and with any radius.

29 The preceding definitions and explanations should be carefully read by the student. Theorems and other definitions will be given in their proper places, which will enable the student to see the foundation upon which the Problems are based. And, in the construction of the Problems, it will be necessary to observe the following rules:—

- 1.—Avoid drawing unnecessary lines, and all which are drawn should be made long enough not to require producing.
- 2.—In determining a point by the intersection of two lines, or two arcs of circles, they should, if possible, cross each other at right angles.
- 3.—In describing circles, hold the compasses as lightly as possible, and as upright as practicable; yet so as to avoid making holes in the paper.
- 4.—In solving problems, draw *given lines* thin and continuous; *lines of construction*, thin and dotted, or barred; and those which constitute the *solution* of the problem, thick and continuous.
- 5.—Do not pass over any problem until you perfectly *understand* its solution.

NOTE.—Before commencing the subsequent Problems, the student should provide himself with the following requisite instruments:—1. Drawing-board, 24 inches long by 18 inches broad. 2. T square, same length as the drawing-board. 3. Two set squares, about 8 inches long, called "Angle 45°," and "Angle 60° and 30°." 4. A pair of "compasses," with shifting leg, pen and pencil legs to fit the same. 5. Drawing-pen, for inking in straight lines. 6. A "protractor," for laying-off angles. 7. A scale of "inches," with the inches subdivided into "tenths," for the measurement of given lines.

PRACTICAL PLANE GEOMETRY.

PLATE III.

PROBLEMS.

30 PROBLEM I.—TO BISECT A GIVEN LINE, THAT IS, TO DIVIDE IT INTO TWO EQUAL PARTS.

Let **A B** (Fig. 1) be the given line (say 1·7" long—the accents indicate inches).

- 1.—From **A**, as a centre, with any radius greater than half of **A B**, describe an arc on each side of the line, as **m** and **n**.
- 2.—From **B**, as a centre, with the same radius, describe arcs cutting the former in **m** and **n**.
- 3.—Through the points of intersection, draw the line **m n**, and it bisects the given line in the point **C**, as was required.

31 The two points **m** and **n** are (by const.) equidistant from the points **A** and **B**, the extremities of the given line : consequently, any point located in the line **m n**, is always equidistant from the two given points **A** and **B**. Hence, the line **m n**, produced indefinitely, is the locus of the centres of all circles whose circumferences pass through the two given points **A** and **B**, that is, any point being taken in the line **m n**, as a centre, with a radius = to the distance from the assumed point to **A** or **B**, a circle may be drawn, whose circumference passes through **A** and **B**.

32 PROBLEM II.—TO BISECT A GIVEN ANGLE.

Let **B A C** (Fig. 2) be the given angle.

- 1.—From the angular point **A**, as a centre, and with any radius, describe an arc cutting the sides **A B**, **A C** in the points **m** and **n**.
 - 2.—From **m** and **n**, as centres, with the same or any other convenient radius, describe arcs cutting each other in **p**. Then, a line drawn from **A**, through the point of intersection **p**, will bisect the angle, as was required.
- 33 As the line **A p** bisects the angle **B A C**, all points located in it are equidistant from the sides of the angle : therefore, the bisecting line of any angle, is the locus of a point which is always equidistant from two given lines which are inclined to one another. Hence (Def. 15), it is the locus of the centres of all circles which touch the two given lines **A B**, **A C**.
-

34 PROBLEM III.—TO DRAW A PERPENDICULAR TO A GIVEN LINE, FROM A GIVEN POINT IN THE SAME.

CASE I.—When the given point is at or near the middle of the given line.

Let **A B** (Fig. 3) be the given line, and **P** the given point.

- 1.—Set off on each side of **P** equal distances, as **P m**, **P n**.
- 2.—From **m** and **n**, as centres, with any radius greater than **P m** or **P n**, describe arcs cutting each other in **q** : and from **P** draw a line through the point of intersection, which will be the required perpendicular.

FIG. 1

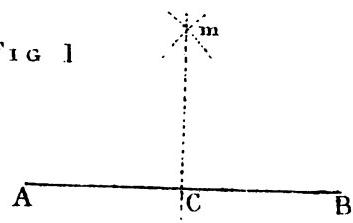


FIG. 2

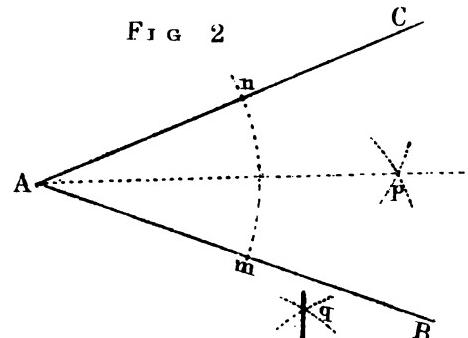


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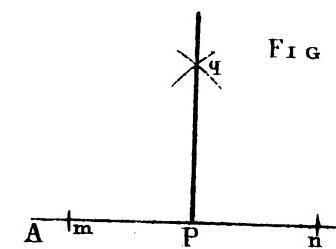


FIG. 4

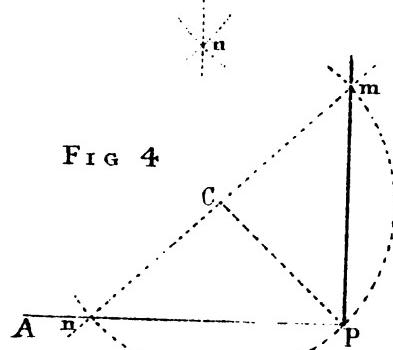


FIG. 5

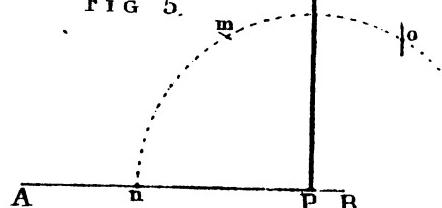


FIG. 6

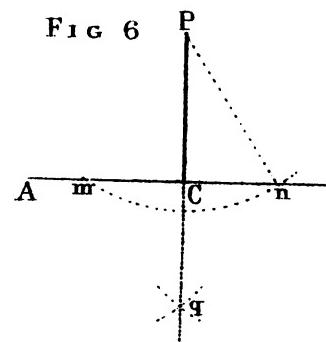


FIG. 7

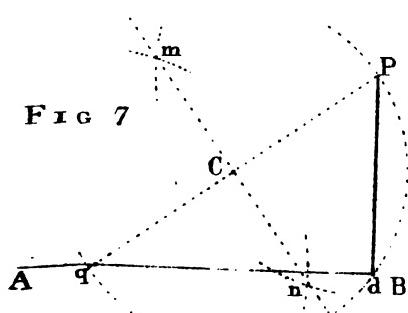


FIG. 8

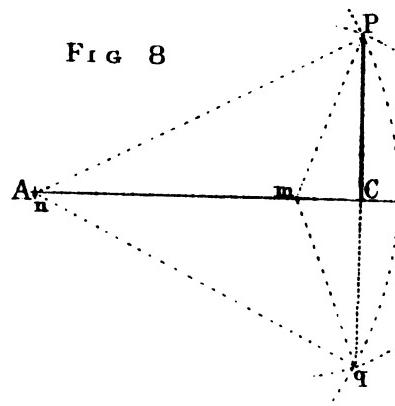
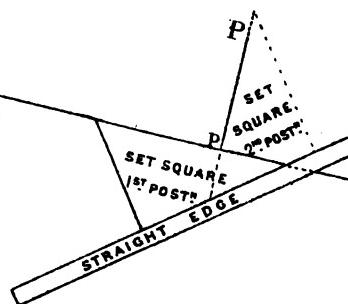


FIG. 9





PRACTICAL PLANE GEOMETRY.

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CASE II.—When the given point is at or near the end of the given line.

Let $\mathbf{A}\mathbf{B}$ (Fig. 4) be the given line, and \mathbf{P} the given point.

- 1.—On that side of the line on which the perpendicular has to be drawn assume any point \mathbf{C} , as a centre, and with the distance $\mathbf{C}\mathbf{P}$, as a radius, describe the arc $\mathbf{m}\mathbf{P}\mathbf{n}$ passing through \mathbf{P} , and cutting $\mathbf{A}\mathbf{B}$ in \mathbf{n} .
- 2.—From \mathbf{n} draw an indefinite line passing through the point \mathbf{C} , and cutting the arc $\mathbf{m}\mathbf{P}\mathbf{n}$ in \mathbf{m} .
- 3.—From \mathbf{P} draw the line $\mathbf{P}\mathbf{m}$, and it will be the required perpendicular.

36

Or thus.

Let $\mathbf{A}\mathbf{B}$ (Fig. 5) be the given line, and \mathbf{P} the given point.

- 1.—From \mathbf{P} , as a centre, with any radius, describe the arc $\mathbf{m}\mathbf{n}\mathbf{o}$.
- 2.—Set off the radius $\mathbf{P}\mathbf{n}$, from \mathbf{n} to \mathbf{m} , and from \mathbf{m} to \mathbf{o} , on the arc $\mathbf{n}\mathbf{m}\mathbf{o}$.
- 3.—From the points \mathbf{m} and \mathbf{o} , as centres, with the same or any other convenient radius, describe arcs cutting each other in \mathbf{q} . And from \mathbf{P} draw a line through \mathbf{q} , and it will be the required perpendicular.

37

PROBLEM IV.—TO DRAW A PERPENDICULAR TO A GIVEN LINE, FROM A GIVEN POINT WITHOUT IT.

CASE I.—When the given point is nearly opposite the middle of the line.

Let $\mathbf{A}\mathbf{B}$ (Fig. 6) be the given line, and \mathbf{P} the given point.

- 1.—From \mathbf{P} , as a centre, describe an arc so as to cut $\mathbf{A}\mathbf{B}$ in two points \mathbf{m} and \mathbf{n} .
- 2.—From \mathbf{m} and \mathbf{n} , as centres, with the same radius or any other convenient radius, describe arcs cutting each other in \mathbf{q} .
- 3.—Through the points \mathbf{P} and \mathbf{q} draw the line $\mathbf{P}\mathbf{c}\mathbf{q}$, which will be the perpendicular required.

38

CASE II.—When the given point is opposite, or nearly opposite, the end of the line.

Let $\mathbf{A}\mathbf{B}$ (Fig. 7) be the given line, and \mathbf{P} the given point.

- 1.—Take any point as \mathbf{q} in the line $\mathbf{A}\mathbf{B}$, and join $\mathbf{P}\mathbf{q}$.
- 2.—Bisect $\mathbf{P}\mathbf{q}$ (Prob. 1) in the point \mathbf{c} , and from \mathbf{c} , as a centre, with the radius $\mathbf{c}\mathbf{P}$ or $\mathbf{c}\mathbf{q}$, describe the arc $\mathbf{q}\mathbf{d}\mathbf{P}$ cutting $\mathbf{A}\mathbf{B}$ in \mathbf{d} .
- 3.—Join $\mathbf{P}\mathbf{d}$, and it will be the perpendicular required.

39

Or thus.

Let $\mathbf{A}\mathbf{B}$ (Fig. 8) be the given line, and \mathbf{P} the given point.

- 1.—Take any two points in $\mathbf{A}\mathbf{B}$, as \mathbf{m} and \mathbf{n} .
- 2.—From \mathbf{m} and \mathbf{n} , as centres, with the distances $\mathbf{m}\mathbf{P}$, $\mathbf{n}\mathbf{P}$ as radii, describe arcs cutting each other in \mathbf{q} .
- 3.—From \mathbf{P} draw the line $\mathbf{P}\mathbf{c}\mathbf{q}$, and it will be the required perpendicular.

40

THEOREM.—The perpendicular is the shortest line that can be drawn from a given point to a given line.

41 Perpendicular lines may be drawn with greater expedition by means of a "straight edge" and "set square," in the following manner, thus :—

Let $\mathbf{A}\mathbf{B}$ (Fig. 9) be the given line, \mathbf{P} or \mathbf{p} the given point.

Place the longest side of the set square so as to coincide with $\mathbf{A}\mathbf{B}$. Then, holding it firm with one hand, place a straight edge or flat ruler against one of the other two sides. Keeping the ruler fixed in this position, move the set square so as to bring its third side upon the ruler. Still keeping the ruler fixed, slide along the set square until its longest side passes through the given point \mathbf{P} or \mathbf{p} . Then, holding the set square fixed, draw the line $\mathbf{P}\mathbf{p}$, which will be the required perpendicular.

PLATE III.

42 PROBLEM V.—TO DRAW A LINE PARALLEL TO A GIVEN LINE, AND AT A GIVEN DISTANCE FROM IT.

Let **A B** (Fig. 10) be the given line, and **c** the given distance (say '7").

- 1.—In **A B**, take any two points as far apart as possible, as **m** and **n**.
- 2.—From **m** and **n** draw **n p**, **m o** perpendicular to **A B** (Prob. 3).
- 3.—From **m** and **n**, as centres, with a radius equal to the given distance **c**, describe arcs cutting the perpendiculars in **o** and **p**.
- 4.—Draw the line **D E** passing through **p** and **o**, and it will be the required parallel.

43 The two points **p** and **o** are (by const.) equally distant from the line **A B**; and, consequently, all points located in **D E** will be at the same distance from the line **A B** (Def. 9). Hence the line **D E** is the locus of a point which is always equidistant from **A B**; and, therefore, it is also the locus of the centres of all circles of the same radii which touches the line **A B**.**44 PROBLEM VI.—TO DRAW A LINE THROUGH A GIVEN POINT PARALLEL TO A GIVEN LINE.**

Let **P** (Fig. 11) be the given point, and **A B** the given line.

- 1.—From **P**, as a centre, with as large a radius as possible, describe the arc **m n** cutting **A B** in **n**.
- 2.—From **n**, as a centre, with the same radius, describe the arc **P q**, meeting **A B** in **q**.
- 3.—From **n**, as a centre, with the distance **P q** as radius, describe an arc cutting **m n** in **m**. Then a line drawn through **P** and **m** will be the required parallel.

45 Parallel lines may readily be drawn, by the aid of a "straight edge" and "set square," thus:—

Let it be required to draw the line **p q** (Fig. 12) parallel to the given line **T S**, either at the given distance **q s**, or through the given point **p**.

Place one side of the set square, as **T g**, coincident with the given line **T S**. Holding the set square firm in this position, place a straight edge against the side **f g** of the set square. Keeping now the straight edge fixed, slide along the set square until the side **T g** passes through **p** or **q**. Then a line drawn through **p** or **q** will be the parallel required.

A little practice will soon enable the student to draw lines, either perpendicular or parallel, with facility.

FIG. 10

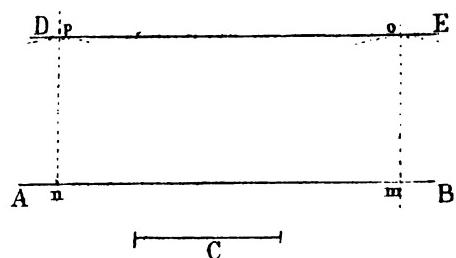


FIG. 11

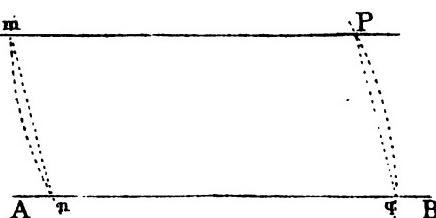


FIG. 12

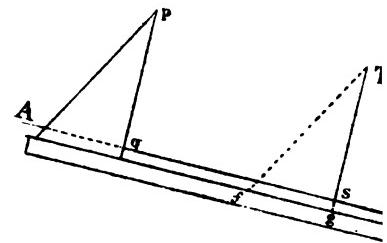


FIG. 13

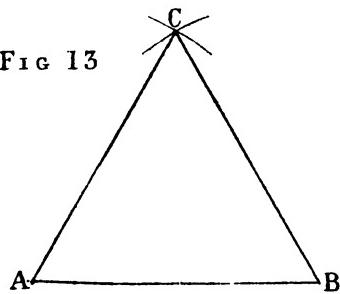


FIG. 14

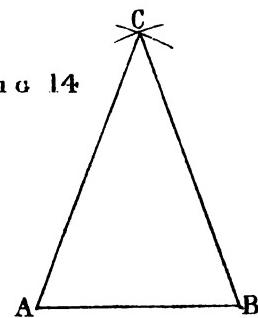


FIG. 15

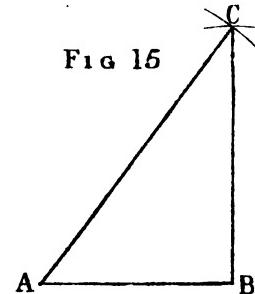


FIG. 16

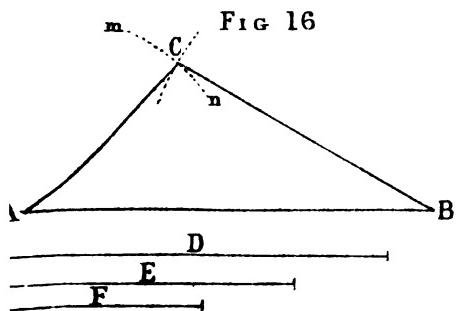


FIG. 17

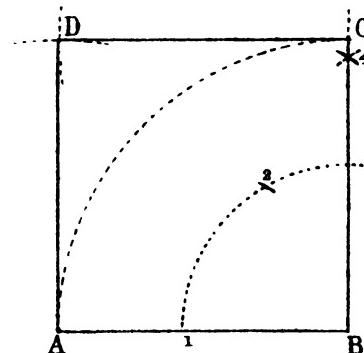
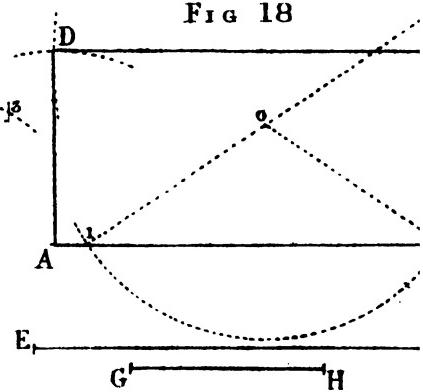


FIG. 18





46 DEF.—Triangles are plane surfaces, bounded by three lines meeting together at their extremities, so as entirely to enclose a space ; and are named either from the character of their angles, or from the proportion of their sides.

47 DEF.—A triangle which has all its sides equal, is called an *equilateral triangle* ; as **A B C** (Fig. 13).

48 PROBLEM VII.—ON A GIVEN LINE TO CONSTRUCT AN EQUILATERAL TRIANGLE.

Let **A B** (Fig. 13) be the given line (say 1·5" long).

- 1.—From **A** and **B**, as centres, and with **A B** as radius, describe arcs cutting each other in **C**.
- 2.—Join **A C**, **B C**, and **A B C** will be the triangle required.

49 DEF.—A triangle which has only two of its sides equal, is called an *isosceles triangle* ; as **A B C** (Fig. 14), having the side **A C = B C**.

50 DEF.—The base of any triangle is that side on which it is supposed to stand ; the vertex is the angular point opposite the base, and the altitude or height is the perpendicular falling on the base, or base produced from the vertex.

51 DEF.—A triangle which has one of its angles a right angle, is called a *right-angled triangle* ; as **A B C** (Fig. 15), where the angle at **B** is a right angle. The side **A C** opposite the right angle is called the *hypotenuse*.

52 PROBLEM VIII.—GIVEN THE BASE AND PERPENDICULAR OF A RIGHT-ANGLED TRIANGLE TO CONSTRUCT IT.

Let the base be = three units in length, and the perpendicular to four of the same kind.

- 1.—Draw the line **A B** (Fig. 15) equal to the given length.
- 2.—From **B** draw **B C** perpendicular (Problem III.) to **A B**, and equal to four units of the same kind as the base.
- 3.—Join **A C**, then **A B C** will be the triangle required.

53 DEF.—A triangle, as **A B C** (Fig. 16), which has all its sides unequal, is called a *scalene triangle*.

54 THEOREM.—Any two sides of a triangle are together greater than the third side.

55 PROBLEM IX.—TO CONSTRUCT A TRIANGLE WHOSE SIDES ARE EQUAL TO THREE GIVEN LINES.

Let **D**, **E**, and **F** (Fig. 16) be the given lines.

- 1.—Draw the line **A B** equal to one of the given line, as **D**.
- 2.—From **A**, as a centre, with a distance equal to one of the other given lines, as **E**, describe the arc **m n** ; and from **B**, as a centre, with the remaining line **E** as radius, describe an arc, cutting **m n** in **c**.
- 3.—Join **C A**, **C B**, and **A B C** will be the triangle required.

56 The preceding Problem is useful in enabling us to locate a point, when its distances from two other points are known. Thus, for example, let us take two points **A** and **B** (Fig. 16), 2" apart, and let it be required to locate a point **C**, 1" from **A**, and 1·5" from **B**.

- 1.—From **A**, as a centre, and with a radius=1", describe the arc **m n**. Then (from Arts. 15 and 24) it is evident that the required point lies in this arc.
- 2.—From **B**, as a centre, and with a radius=1·5", describe an arc cutting **m n** in **c**, which will be the position of the point **C**, since all points in the second arc are 1·5" from **B** (Arts. 15 and 24); and it has been shown that all points in the arc **m n** are 1" from **A**. Hence, where the two arcs intersect is the location of the required point.

If we use both of the above radii at *each* of the points **A** and **B**, and describe arcs with each radius on both sides of **A B**, four points can be located at the given distances from the given points.

57 DEF.—A quadrilateral figure is that which is contained or bounded by four right lines.

58 DEF.—A quadrilateral figure which has its opposite sides equal and parallel is called a *parallelogram*.

59 DEF.—A parallelogram which has all its sides equal, and all its angles right angles, is termed a *square*.

Let **A B** (Fig. 17) be the given line (say 1·5" long).

- 1.—From one end of the line, as **B**, draw an indefinite line perpendicular to **A B** (Problem III.).
- 2.—Make **B C=B A**, and from **C** and **A**, as centres, with **A B** as radius, describe arcs cutting each other in **D**.
- 3.—Join **A D**, **C D**, and **A B C D** will be the required square.

61 DEF.—A parallelogram which has its adjacent sides unequal, and its angles, right angles, is termed an *oblong* or *rectangle*.

Let **E F** and **G H** (Fig. 18) be the given lines.

- 1.—Make **A B=E F**, and from **B** draw the indefinite line **B 2** perpendicular to **A B** (Problem III.).
- 2.—From **B** set off **B C=G H**, and from **A**, as a centre with the distance **B C** or **G H**, describe an arc, as at **D**.
- 3.—From **C**, as a centre, with a radius=**A B** or **E F**, describe an arc cutting the former in **D**.
- 4.—Join **A D**, **C D**, and **A B C D** will be the rectangle required.

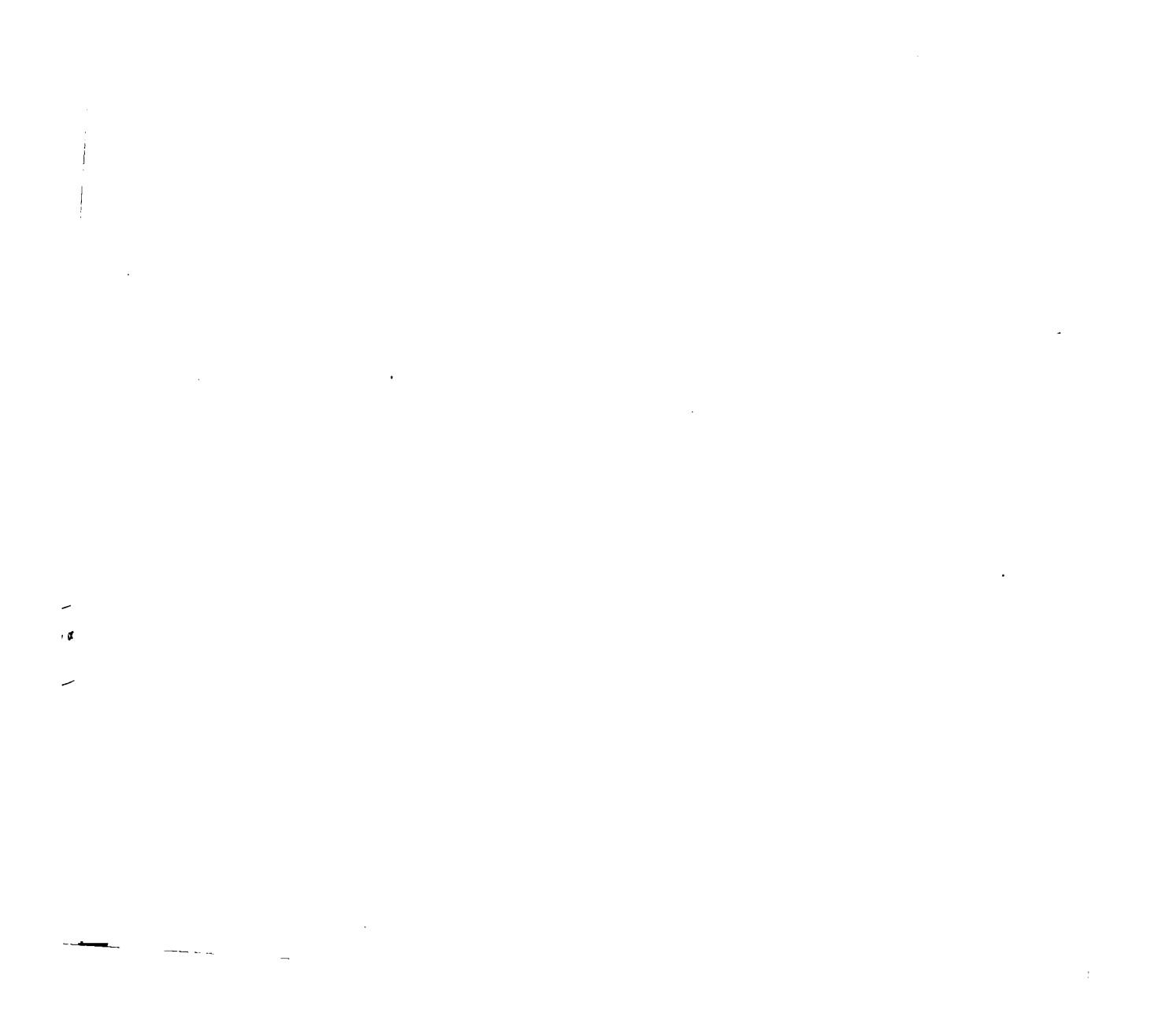


FIG 19

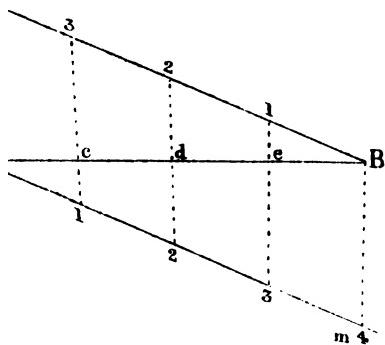


FIG 20

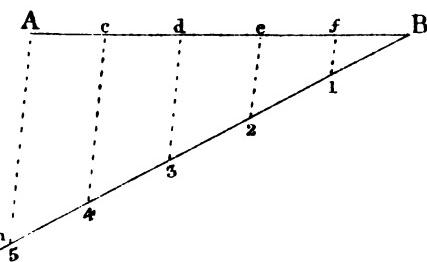


FIG 21

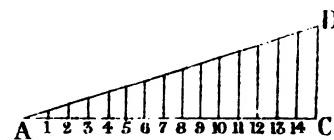


FIG 22

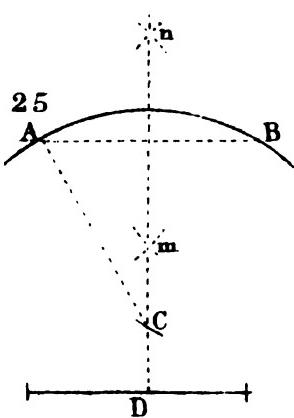
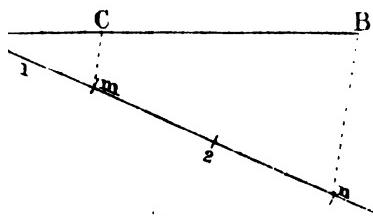


FIG 23

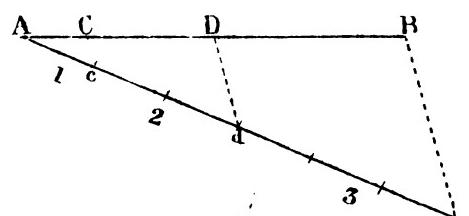


FIG 24

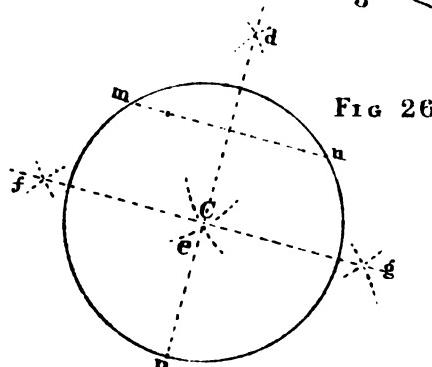
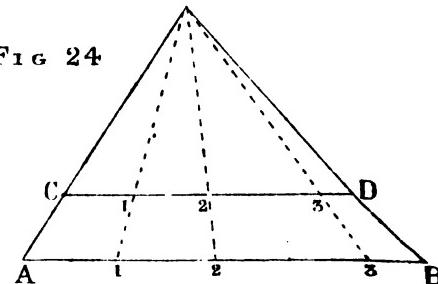


FIG 26

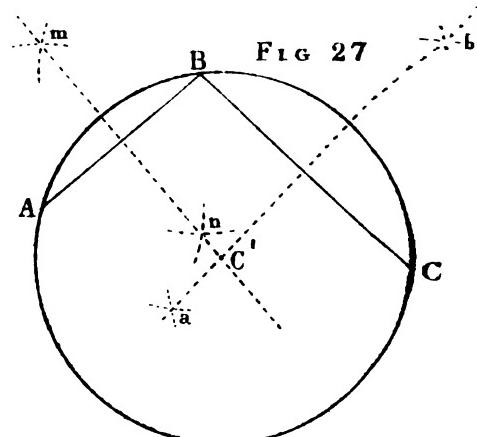


FIG 27

PLATE IV.

63 THEOREM.—If a straight line be drawn parallel to any side of a triangle, it shall cut the other sides, or those sides produced proportionally.

64 PROBLEM XII.—TO DIVIDE A GIVEN LINE INTO ANY PROPOSED NUMBER OF EQUAL PARTS.

Let **A B** (Fig. 19) be the given line (say 2" long) which is required to be divided into (say) four equal parts.

- 1.—From **A** draw an indefinite line, as **A' 4'**, making any angle with **A B**.
- 2.—From **B** draw an indefinite line parallel to **A' 4'** (Problem VI.).
- 3.—On each of these lines, commencing at the points **A** and **B**, set off as many equal distances as the line **A B** is to be divided into (in this case four), as **A 1', 1' 2', 2' 3', 3' 4'**, and **B 1, 1 2, 2 3, 3 4**.
- 4.—Join the points **A 4, 1' 3, 2' 2, 3' 1**, and **4' B**. Then **A B** will be divided into four equal parts, as was required.

65 The above Problem may be simplified in its construction, as follows, for example:—

Let it be required to divide a given line **A B** (Fig. 20), 2" long, into (say) five equal parts.

- 1.—From either end of the given line, draw an indefinite line, making any angle with it, as **B 5**: and on this line, commencing at **B**, set off five equal parts; as **B 1, 1 2, 2 3, 3 4, 4 5**.
- 2.—Join **A 5**, and through the points **4, 3, 2, 1** draw lines parallel to **A 5**, by the aid of the set square and straight edge (as shown in Art. 45), meeting **A B** in the points **c, d, e, f**. Then the line **A B** will be divided into five equal parts, as was required.

66 PROBLEM XIII.—TO DIVIDE A GIVEN LINE INTO TWO PARTS, IN A GIVEN RATIO.

Let **A B** (Fig. 22) be the given line, and the given ratio **p** to **q**, where **p** and **q** may be any numbers, whole or fractional. In this case, let **p=1**, and **q=2**.

- 1.—From **A** draw an indefinite line, making any angle with **A B**, as **A n**.
- 2.—Set off from **A** on **A m** any distance, as **A m**. Taking **A m** as a unit, repeat it twice from **m** to **n**. Then it is evident that **A n** is divided into two parts in the point **m**; so that **A m : m n :: 1 : 2**. Therefore, in order to divide **A B** in the same ratio, join **B n**; from **m** draw **m c** parallel to **B n** (Art. 43). Then **A B** is divided in the point **c**; so that **A C : C B :: p : q :: 1 : 2**, as was required.

PRACTICAL PLANE GEOMETRY.

- 67 In the same manner a line may be divided into any number of unequal parts proportionally. For example, divide a line **A B**, 2' long, into three parts in the proportion of 1, 2, and 3.

Let **A B** (Fig. 23) be the given line, 2' long.

- 1.—From **A** draw the indefinite line **A b**, making any angle with **A B**.
- 2.—From **A**, on **A b**, set off any distance **A c**; repeat it twice from **c** to **d**, and three times from **d** to **b**.
- 3.—Join **B b**, and from **d** and **c** draw **d D**, **c C** parallel to **B b**. Then **A B** will be divided into the points **C** and **D**; so that **A c**, **C D**, and **D B** are in the proportion of 1, 2, and 3, as required.

- 68 **THEOREM.**—Diverging lines cut parallel lines proportionally.

- 69 **PROBLEM XIV.**—TO DIVIDE ANY GIVEN SHORT LINE, AS A UNIT, INTO ANY FRACTIONAL NUMBER OF PARTS.

Let **C D** (Fig. 21) be the given line, and let it be required to divide it into say fifteenths.

- 1.—From **c** draw the indefinite line **C A**, making any angle with **C D**.
- 2.—Take any distance in the compasses, and commencing at **C**, set off as many distances as **C D**, the given line, is to be divided into fractional parts (in this case fifteen).
- 3.—Figure the last but one from **C**, 1 and each successive distance up to **C**, as shown in the diagram.
- 4.—Join **A D**. Through the points 1, 2, 3, &c., draw lines parallel to **C D** (Art. 45) till they meet **C D**. Then these parallel lines are fractional parts of **C D**. For instance, that marked $1 = \frac{1}{15}$ of **C D**; $2 = \frac{2}{15}$, $3 = \frac{3}{15}$ or $\frac{1}{5}$, and so on, each parallel line from **C D** decreasing by $\frac{1}{15}$. Consequently, on this principle a *very short* line may be divided into any number of parts. And it is on this principle the Diagonal Scale is constructed, as will be shown in a future Part.

- 70 **PROBLEM XV.**—TO DIVIDE A GIVEN LINE PROPORTIONALLY TO A GIVEN DIVIDED LINE.

Let **A B** (Fig. 24) be the given line, and **C D** the given divided line.

- 1.—Draw **A B** parallel to **C D**.
- 2.—Join **A C**, **B D**, and produce the lines till they meet in **E**.
- 3.—From **E** draw lines through the points of division, 1, 2, 3, in **C D**, till they cut **A B** in the points **1'**, **2'**, and **3'**. Then **A B** is divided proportionally, as required.

NOTE.—When the two lines are nearly of the same length, it would be better to use Problem XIII.

71 PROBLEM XVI.—TO DESCRIBE THE CIRCUMFERENCE OF A CIRCLE, OR ARC OF A CIRCLE, PASSING THROUGH TWO GIVEN POINTS, AND WITH A GIVEN RADIUS.

Let **A** and **B** (Fig. 25) be the given points (say 1·1" apart), and **D** the given radius (say 1·2").

From what has been stated (in Art. 31) the centre of the required circle will lie in the line **m n**, which bisects perpendicularly the distance between the given points. Then, from **A** or **B**, as a centre, and with the given radius, describe an arc cutting **m n** in **c**, which will be the required centre. Therefore, from centre **c**, with the radius of the given circle, describe the arc or the circle as required. But the most practical construction would be as follows:—

- 1.—From **A** and **B**, as centres, and with a radius = **D** (1·2"), describe arcs cutting each other in **O**.
- 2.—From **O**, as a centre, and the same radius, describe the arc **A B**, which will pass through the given points.

72 THEOREM.—If a chord of a circle be bisected perpendicularly, the bisecting line passes through the centre of the circle.

73 PROBLEM XVII.—TO FIND THE CENTRE OF A GIVEN CIRCLE.

The solution of this Problem becomes self-evident from the above Theorem. Thus : in the given circle **m n p** (Fig. 26), take any chord, as **m n**. Then (Theor. Art. 72) the line which bisects this chord prependicularly, will pass through the required centre ; and, therefore, the intercepted part **p q** will be a diameter, the bisection of which gives the required centre **C**.

74 PROBLEM XVIII.—TO DESCRIBE A CIRCLE THE CIRCUMFERENCE OF WHICH SHALL PASS THROUGH THREE GIVEN POINTS NOT SITUATED IN THE SAME STRAIGHT LINE.

The solution to this Problem depends on the same principles as the preceding. Thus : let **A**, **B**, and **C** (Fig. 27) be the given points. Join **A B**, **B C**; then the lines **A B** and **B C** are chords of the same circle, of which the bisecting line of each passes through the centre of the required circle. Hence the point **O** where the bisecting lines **m n**, **a b** cut each other is the required centre. Therefore, from **O**, as a centre, and with a radius = to the length of any one of the three given points from **O**, describe the circle **A B C**, which will pass through the points **A**, **B**, and **C**, as required.

QUESTIONS AND EXAMPLES FOR PRACTICE ON THE FOREGOING DEFINITIONS AND PROBLEMS.

NOTE.—The questions marked with an asterisk (*) are taken (by permission) from the Examination Papers of the Government Department of Science and Art.

1. How many different kinds of lines are there ? Give an example of each.
2. Define a surface. How many kinds of surfaces are there ? By what general rule are they distinguished from each other ?
3. Define parallel lines, and give an example. Is it necessary that parallel lines should always be straight lines ?
4. What is an angle ? How are angles expressed ?
5. Does the magnitude of an angle depend upon the length of the lines by which it is formed ?
6. Explain the terms right-angle, acute-angle, obtuse-angle, and perpendicular.
7. Define the terms circle, circumference, diameter, radius, arc, chord, and tangent.
8. What is meant by a "geometrical locus" ? Trace a line, such that every point in it shall be 1 inch from a given point.
9. Bisect a line 3·5" long ; and show that all points which are equally distant from the extremities of the given line lie in the bisecting line.
10. Two lines, each 3" long, meet at a point ; their other extremities are 2" apart ; find the locus of a point that is equally distant from the two given lines.
- 11*. Step 14 equal distances of ·3 of an inch along a line, and from every alternate point of division as a centre, describe a semicircle of ·3 of an inch radius, to lie alternately on opposite sides of the line.
- 12*. Draw 8 concentric circles at 1 of an inch apart, the largest to be 1·5 inches radius ; put a light tint of Indian ink between the alternate rings formed by them.
13. Locate a point which shall be 2" distant from the extremities of a line 2" long ; through the point draw a line parallel to the given line.
14. Trace a line, such that every point in it shall be 1·5 from a given line.
15. Define a triangle. By what names are triangles distinguished according to their form ? Give an example of each kind.
16. Does the magnitude of a triangle depend upon its sides, or its angles ?
17. Draw a triangle whose sides are 2", 3", and 2·8" long. Circumscribe this triangle by a circle.
18. Two points, A and B, are 3" apart; a third point, C, is 2·6" from A and 3·5" from B; find a fourth, D, which shall be equidistant from the three given points, A, B, and C.
- 19*. Determine two points, P and Q, on the same side of an indefinite line, A B, at 1" and 1·5" from it, and at 2" from each other ; find a point C in A B equidistant from P and Q.
20. Make a square of 2·5 sides ; and on each side of it an equilateral triangle.
21. Make a rectangle 3" long and 1" broad.
- 22*. Draw a triangle having its sides 2·5", 2·75", and 3" long, with the inscribed and circumscribed circles.
23. Divide a line A B = 3·3" long, in the point C, so that A C : B C : : 3 : 5.
24. Divide a line 5·5" long into three parts, in the proportion of 3, 4, and 5 ; construct a triangle whose sides are equal to the three segments.
25. Locate a point C, 3" and 4" distant from the extremities of a line A B 5" long ; and determine the length of the shortest line which can be drawn from C to meet A B.
26. Divide a line 4·5" long into 21 equal parts.
27. Find the tenth part of a line 3 of an inch long.
28. Find the seventeenth part of a line 5 of an inch long.
29. On a line 2·5" long, construct an equilateral triangle ; on each side of it construct a square ; circumscribe the whole by a circle.
30. Draw 16 parallel lines ·3 of an inch apart, and 16 others at right angles to the former at the same distances : put a light tint of Indian ink in every alternate square, so as to form a chequer pattern.

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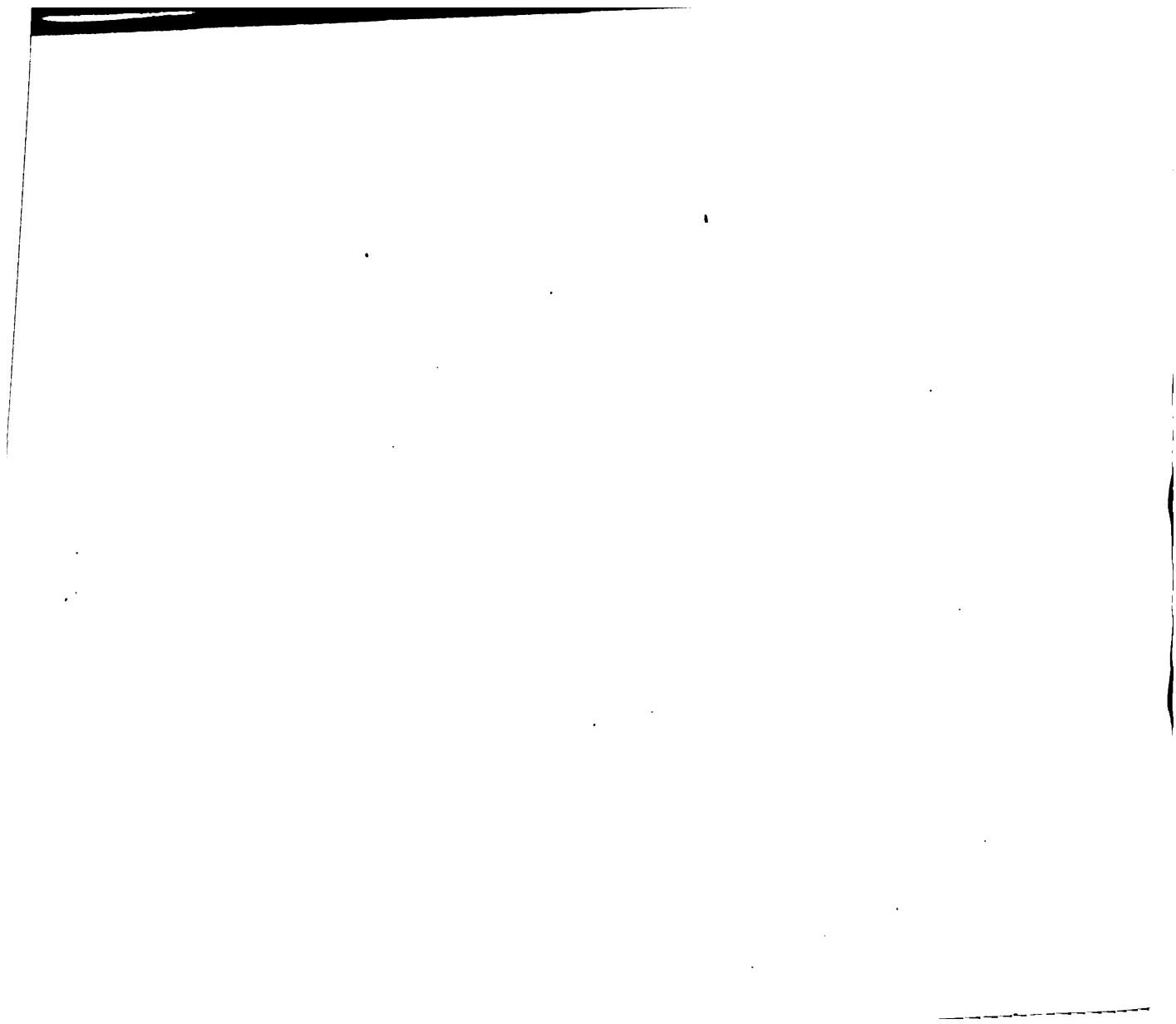
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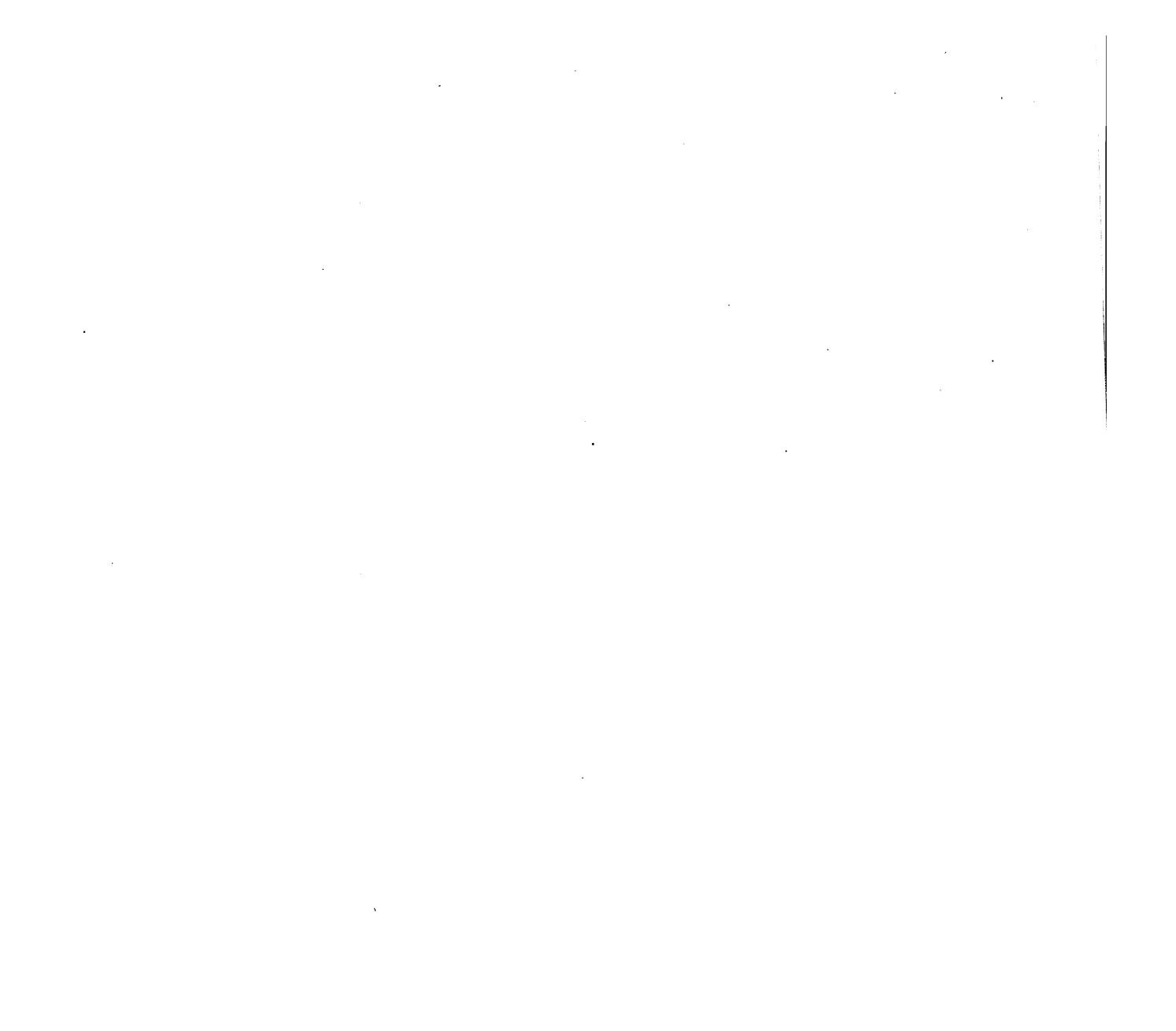
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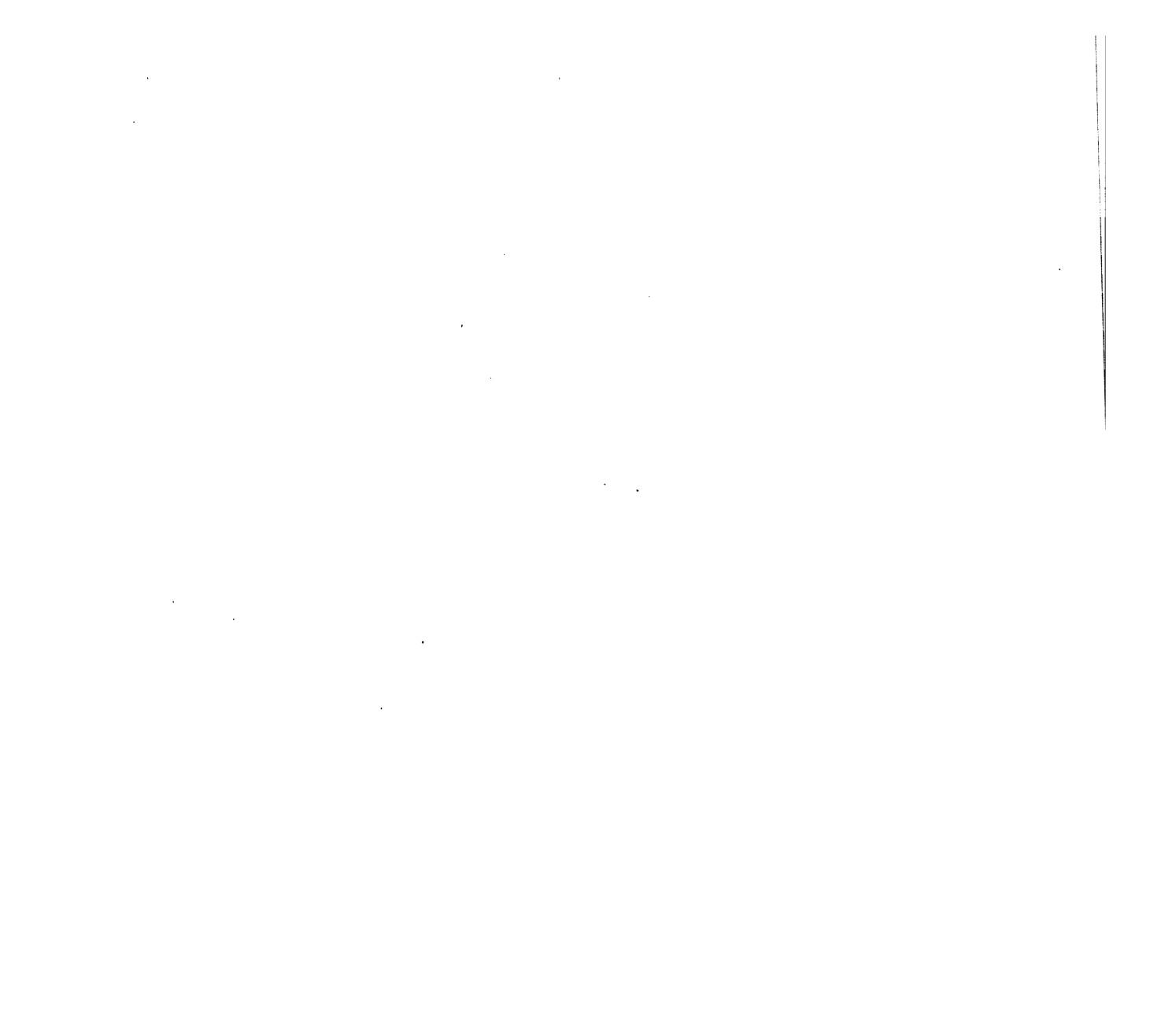
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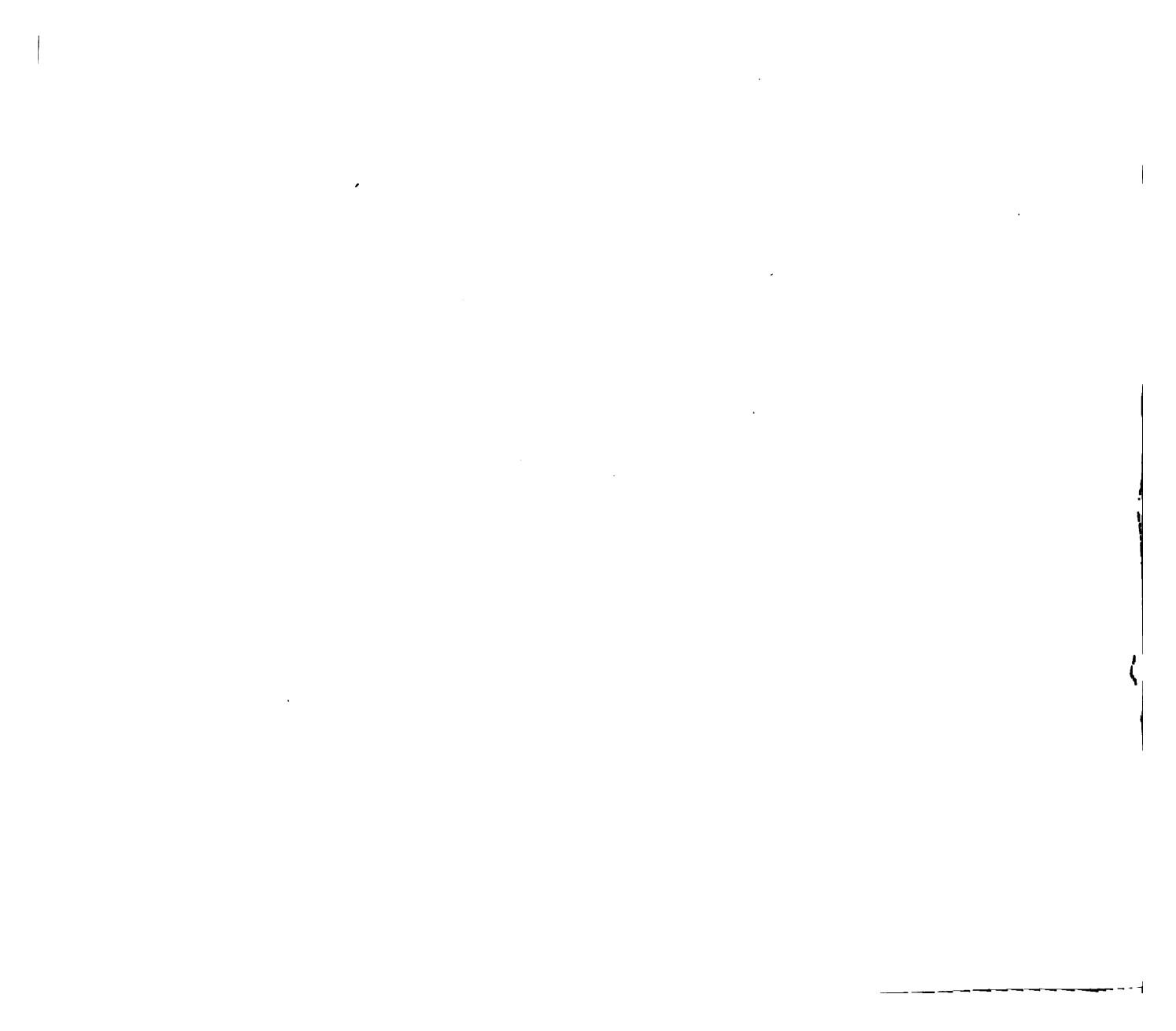






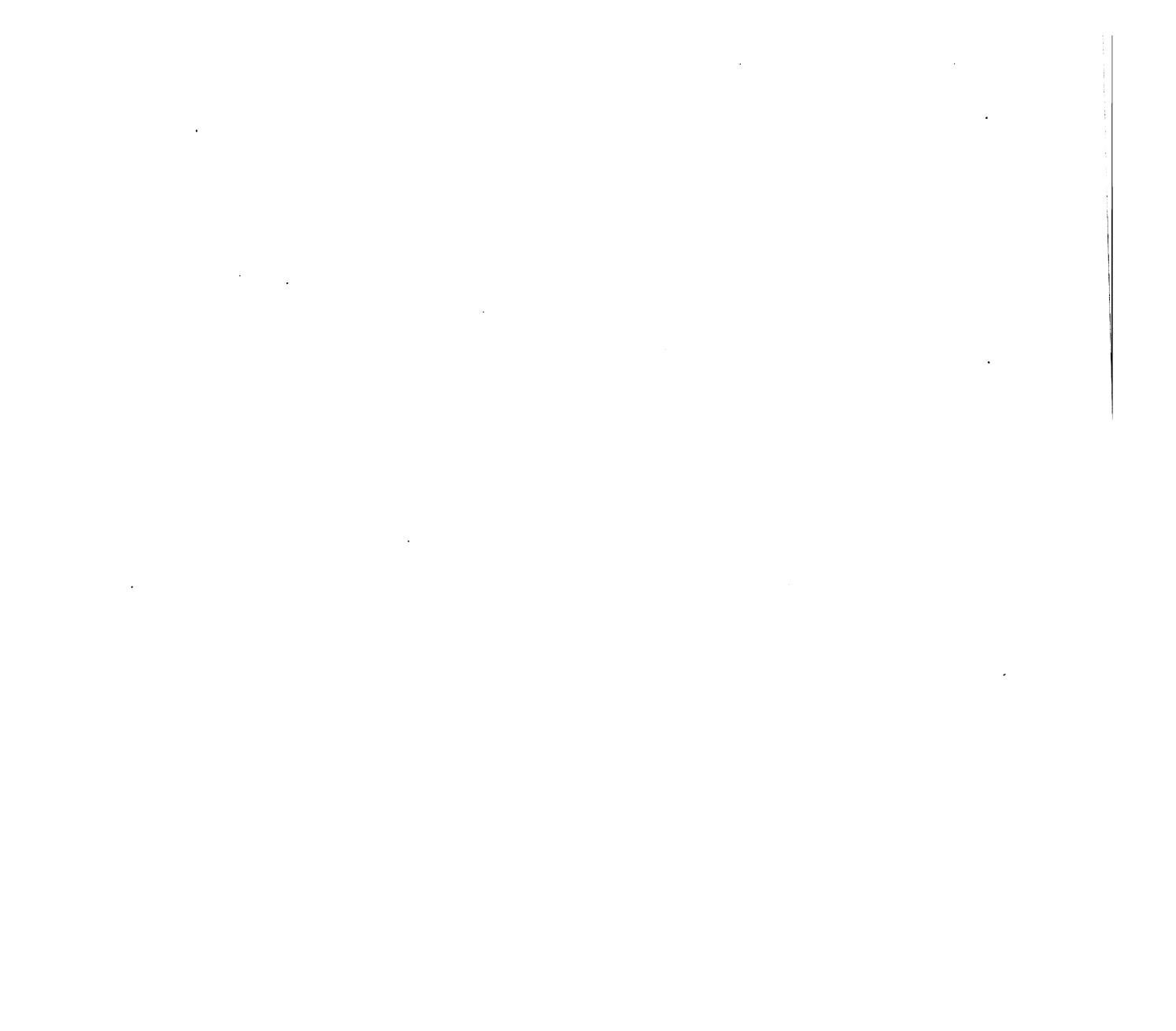


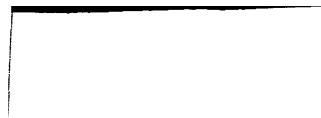












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